Optimum Heat Rejection Temperatures for Spacecraft Heat Pumps

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Use of two heat rejection temperatures instead of one for a heat-driven heat pump is shown to reduce spacecraft waste heat radiator area and heat rejection system mass. The optimum high rejection temperature T_1 is shown to be somewhat less than Kerrebrock's rule, three-fourths of the heat source temperature T_H. The optimum low rejection temperature T_2 is 50–80 K above the cooling load temperature T_C. General results are presented based upon the assumption of constant fractions of Carnot efficiency for the heat engine and refrigerator, and specific results are reported for a mercury-vapor engine coupled to an isobutane-R113 binary refrigerator. A single dimensionless grouping composed of ratios of equipment mass per unit power to radiator mass per unit area, and the ratios of the sink temperature T_S and cooling load temperature T_C to T_H, are shown to be the major parameters characterizing the advantages of a spacecraft heat pump.

Nomenclature

A	= radiator area, m^2
a	= radiator mass per unit area, kg/m^2
B	= equipment to radiator parameter, b/a, m^2/W
b	= equipment mass per unit power, kg/W
C	= constants in nonlinear models, Eqs. (19) and (20)
c	= constant in mass relationship, Eq. (6), kg
h	= specific enthalpy, J/kg
M	= mass, kg
P	= pressure, N/m^2
Q	= heat flow, W
s	= specific entropy, J/kg K
T	= temperature, K
W	= mechanical power or heat flow of high availability, W
ϵ	= radiator emissivity
ζ	= Carnot efficiency or COP
η	= component isentropic efficiency
σ	= Stefan-Boltzmann constant
ϕ	= fraction of Carnot efficiency or COP

Subscripts

C	= cold plate or cooling load
c	= refrigerator compressor
E	= engine
e	= engine expander
H	= high temperature heat source
R	= refrigerator
r	= refrigerator expander
S	= sink or passive system rejecting to sink
1	= engine radiator
2	= refrigerator radiator

Introduction

HEAT removed in the cooling of spacecraft equipment is ordinarily radiated to space at nearly the equipment temperature. Only small temperature drops are needed to transfer the heat through heat pipes to the radiator. Gas-controlled heat pipes use noncondensable gas to block or open up radiator area as necessary to maintain the cold plate temperature at T_C when the heat load to be radiated varies. The option exists, however, to use a heat pump to receive the heat load at temperature T_C and reject it at a higher temperature, thus enabling the radiator to be much smaller. This option appears attractive[1,2] for spacecraft or space stations with high power equipment and a high temperature heat source or a power source to drive the heat pump.

A heat-driven heat pump (HDHP) uses heat Q_H at high temperature T_H to provide available energy so that load Q_C at temperature T_C can be boosted in temperature. It has been assumed that the sum $Q_H + Q_C$ plus whatever work is required is to be rejected at a single temperature T_1. Kerrebrock[3] showed that $T_1 = (3/4)T_H$ is the optimum heat rejection temperature when the HDHP is ideal, i.e., when a Carnot engine drives a Carnot refrigerator. Kerrebrock showed that, even when the device operates at half of Carnot efficiency and $T_1 = (3/4)T_H$, the radiator area is considerably reduced as long as $T_H > 3T_C$.

For a space vehicle, the system mass including the radiator mass is a prime concern. Dexter and Haskin[2] found the system mass to be reduced by using a work-driven heat pump (WDHP) when the power rejected was of order 10 kW or greater. For a 100 kW heat rejection system, an optimum T_1 of approximately 60 K (108°F) above T_C was found, on the basis of empirical correlations for refrigerator COP vs lift (T_2–T_C) and engine and refrigerator specific power vs load. Drolen[4] used published specifications of four power systems and analyzed a system driven by both work W and high temperature heat Q_H. The mass of the heat rejection system was minimized by varying the relative amounts of work and heat to drive the heat pump. Drolen found a small savings in mass when T_1 was high enough to reduce the radiator area by a divisor of two.

Grossman[5] compared area- and mass-optimized HDHP and WDHP systems and found similar radiator area reductions for both. System mass reductions were much greater for the HDHP than the WDHP. As a specific realization of an HDHP, a lithium bromide-water absorption unit with a mass/power ratio of 12 kg/kW was shown to reduce radiator area by 37% with no change in spacecraft mass. Interest in the heat pump options is also evidenced in the work of Kobayashi[6] and Peterson.[7]

It is the purpose of this paper to point out that two rejection temperatures, a higher temperature T_1 and a lower T_2, offer mass savings over the use of a single T_1. Moreover, the effect of radiator sink temperature T_S and engine and refrigerator weights are considered. In addition to a general analysis based upon assumed fractions of Carnot efficiency, results of an

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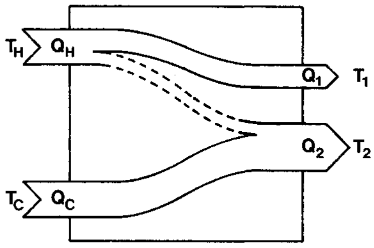

Fig. 1 Heat flow schematic.

analysis of a mercury vapor Rankine heat engine driving an isobutane-Freon 113 binary-cycle reverse-Rankine refrigerator are presented. The results for these high efficiency devices are thought to establish benchmarks against which other systems can be compared.

Analysis

HDHP Operation

Figure 1 depicts the operation of a general HDHP. Heat Q_H enters at high temperature T_H. Within the thermodynamic "black box" one can imagine a heat engine producing work W and rejecting $Q_H - W$ as waste heat Q_1 at temperature T_1. The work W can be imagined driving a refrigerator that accepts heat load Q_C and rejects it, together with additional heat equal to W, as waste heat Q_2 at temperature T_2. In a mechanical system, T_H might be a boiler temperature, T_1 an engine condenser temperature, $Q_H - Q_1$ shaft work from the engine to a compressor, T_2 the refrigerator condenser temperature, and T_C the evaporator temperature. In an absorption system T_H is the refrigerant generator temperature (e.g., the boiling point of a concentrated lithium bromide solution), T_1 is the refrigerant condenser temperature, T_2 is the absorber temperature, and T_C the refrigerant evaporator temperature. With a terrestrial system, one usually uses the same coolant to remove Q_1 and Q_2 so $T_1 = T_2$. With a spacecraft, it is preferable to have T_1 higher than T_2 so the Q_1 radiator can be smaller.

The first law prescribes the exact equality

$$Q_H + Q_C = Q_1 + Q_2 \tag{1}$$

The second law prescribes an inequality, that no device can exceed the efficiency of an ideal reversible machine. Accordingly, one can imagine any reversible cycle, such as a Carnot or Stirling engine driving a Carnot or Stirling refrigerator picking up a load $Q_C = 1$ and find the minimum possible $Q_{H,\min}$ and thus the minimum possible $Q_1 + Q_2 = 1 + Q_{H,\min}$

$$Q_H / Q_C \geq 1/(\zeta_R \zeta_E) \tag{2}$$

where

$$\zeta_R = 1/(T_2/T_C - 1), \qquad \zeta_E = 1 - T_1/T_H \tag{3}$$

System Mass

The mass of each radiator is linear in radiator area. The area is related to the radiator temperature and power by

$$Q_i = \epsilon A_i \sigma (T_i^4 - T_S^4), \qquad i = 1, 2 \tag{4}$$

Various sources of irradiation such as the sun and Earth are accounted for in the effective sink temperature T_S by summing the irradiation-absorptivity products and equating the sum to $\epsilon\sigma T_S^4$. With $M_i = aA_i$ one finds from Eq. (4)

$$M_i = \frac{aQ_i}{\epsilon\sigma(T_i^4 - T_S^4)}, \qquad i = 1, 2 \tag{5}$$

Note that, for simplicity, equal specific masses $a_1 = a_2 = a$, equal emissivities $\epsilon_1 = \epsilon_2 = \epsilon$, and identical sink conditions $T_{S1} = T_{S2} = T_S$ are assumed.

The total mass of the heat rejection system is the mass of the radiators plus that of the heat pump itself. The heat pump is composed of components associated with the heat streams Q_H, Q_1, Q_2, Q_C (e.g., the generator, condensor, absorber, and evaporator) and mechanical components (pumps, engines, compressors). The source of Q_H may be a solar concentrator with absorber or a shielded nuclear reactor, and the added mass needed to produce Q_H may be a burden to the heat pump system. If Q_H is drawn from the waste heat of a power source, a credit may be taken for reducing the power source waste heat radiator; i.e., b_H may be negative. The heat pump mass is assumed to be given by

$$M_{HP} = b_H Q_H + b_C Q_C + b_1 Q_1 + b_2 Q_2 + c \tag{6}$$

Equation (6) is partitioned arbitrarily, because Q_1/Q_H and Q_2/Q_C are close to unity, and Q_C is specified to be unity.

$$M_{HP} = (b_H + b_1 Q_1/Q_H) Q_H + (b_C + b_2 Q_2/Q_C + c/Q_C) Q_C \tag{7}$$

$$M_{HP} = b'_H Q_H + b'_C \tag{8}$$

Parameters B_H and B_C are introduced

$$B_H = b'_H / a, \qquad B_C = b'_C / a \tag{9}$$

The units of B_H and B_C are $m^2 W^{-1}$.

For comparison, the mass of a passive system that radiates Q_C at $T_2 = T_C$ is used. From Eq. (5)

$$M_S = \frac{aQ_C}{\epsilon\sigma(T_C^4 - T_S^4)} \tag{10}$$

Then the ratio of the total heat pump system mass to the passive system mass is

$$\frac{M}{M_S} = \frac{\epsilon(\sigma T_C^4 - \sigma T_S^4)}{Q_C}\left[B_H Q_H + B_C Q_C + \sum_{i=1}^{2} \frac{Q_i}{\epsilon(\sigma T_i^4 - \sigma T_S^4)}\right] \tag{11}$$

$$\frac{M}{M_S} = (1 - T_S^{*4})\left[(B_H^* Q_H^* + B_C^*) \sum_{i=1}^{2} \frac{Q_i^*}{(T_i^{*4} - T_S^{*4})}\right] \tag{12}$$

where

$$T_S^* = T_S/T_C, \qquad T_i^* = T_i/T_C, \qquad i = 1,2 \tag{13}$$

$$B_H^* = \epsilon\sigma T_C^4 B_H, \qquad B_C^* = \epsilon\sigma T_C^4 B_C \tag{14}$$

$$Q_H^* = Q_H/Q_C, \qquad Q_i^* = Q_i/Q_C, \qquad i = 1,2 \tag{15}$$

Allowance for Nonideal Performance

Taking the view that less than ideal devices operate within the thermodynamic "black box" allows one to write

$$Q_2^* = 1 + 1/(\phi_R \zeta_R), \quad Q_1^* = (1 - \phi_E \zeta_E) Q_H^*$$
$$Q_H^* = 1/(\phi_E \phi_R \zeta_E \zeta_R) \tag{16}$$

where parameters ϕ_E and ϕ_R (less than one) are hypothesized engine and refrigerator efficiencies, respectively.

It is clear that with assumed constant values of ϕ $(= \phi_E = \phi_R)$, T_C, T_H, B_C, and B_H, the mass ratio given by Eq. (12) can be minimized with respect to T_1 and T_2.

Rankine Cycle Heat Pump

For real equipment, ϕ_E and ϕ_R are likely to vary somewhat with operating conditions. The Rankine cycle was chosen to explore the variation, although much more detailed models have been established for terrestrial heat pump systems, e.g.[8,9] The Rankine cycle is put forward as a "stocking horse" against which to compare other heat pump systems. Figure 2 shows the engine portion of the cycle on a T-s diagram, and the refrigerator part on a P-h diagram.

Table 1 Heat pump equipment mass

Engine	b'_H, kg/kW	B_H, m^2/kW[a]	Reference
SP100 nuclear power system	1.8	0.18	2
Solar dynamic engine	25	2.5	4
Dynamic isotope engine	20	2.0	4

Refrigerator	b'_C, kg/kW	B_C, m^2/kW[a]	Reference
Vapor compression compressor	0.4	0.04	2
Commercial vapor compression heat pump	2.4	0.24	10

[a]Assuming a nominal radiator area density of $a = 10$ kg/m^2.

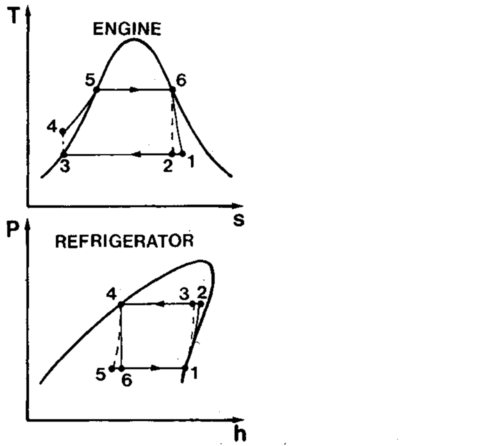

Fig. 2 Engine and refrigerator cycles (dashed line indicates isentropic process).

The engine cycle has isothermal (T_H) heat addition to the boiler followed by a wet expansion with an assumed isentropic expander efficiency $\eta_e \leq 1$. Then isothermal (T_1) heat rejection takes place, followed by an isentropic feedwater pump and a nonisothermal feedwater heater. Irreversible processes are seen to be the less than ideal expansion and the feedwater heating. The ratio of the engine cycle efficiency to the Carnot efficiency ζ_E defines ϕ_E as a function of T_1 and T_H.

The refrigerator is assumed to have a wet compression with an isentropic compression efficiency $\eta_c = 0.8$ followed by isothermal (T_2) heat rejection. Wet compression of saturated vapor is assumed in preference to the conventional choice of dry compression of superheated vapor because wet compression may result in less irreversibility. Wet expansion can take place either through isenthalpic (Joule-Thompson) expansion $\eta_r = 0$ or through an expander of isentropic efficiency $\eta_r = 0.8$. Isothermal (T_c) evaporation follows. Here the compression and expansion are irreversible. Again, the ratio of the cycle coefficient of performance (COP) to the Carnot COP fixes ϕ_R as a function of T_2 and T_C.

With the stocking horse cycle defined in Fig. 2, the relations for ϕ_E and ϕ_R may be expressed in terms of component efficiencies η_c, η_e, and η_r and the operating conditions $T_H = T_{E6}$, $T_C = T_{R1}$, $T_1 = T_{E1}$ and $T_2 = T_{R2}$.

$$\phi_E = \frac{1}{\zeta_E}\left\{\frac{\eta_e(h_{E6} - h_{E2}) - (h_{E4} - h_{E3})}{(h_{E6} - h_{E4})}\right\} \tag{17}$$

$$\phi_R = \frac{1}{\zeta_R}\left\{\frac{(h_{R1} - h_{R6})}{(h_{R3} - h_{R1})/\eta_c - \eta_r(h_{R5} - h_{R4})}\right\} \tag{18}$$

Results

Evaluation of Parameters

Table 1 shows values of b'_H and b'_C cited in the literature.[2,4,10] Values of $a = 5$ to 20 kg/m^2 for the radiators are commonly assumed.[2,4,5] Based upon these values and judgment, "low," "nominal," and "high" cases were chosen for examination as shown in Table 2.

Table 2 Range of parameters investigated

Parameter	Low value	Nominal value	High value
Temperatures			
T_H(K)	700	800	900
T_C(K)	270	300	330
T_S(K)	100	200	250
Mass parameters			
B_C(m^2/kW)	0.01	0.10	1.00
B_H/B_C	⅓	1	3
Constant efficiencies			
ϕ_E	0.3	0.6	1.0
ϕ_R	0.3	0.6	1.0
Expander efficiencies			
η_e	0.6	0.8	—
η_r	—	0	0.8
Radiator emissivities			
ϵ	—	0.8	—

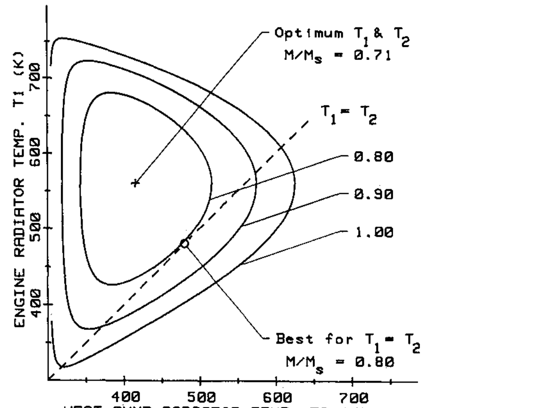

Fig. 3 Mass ratio contours for constant engine and refrigerator efficiencies ($\phi_E = \phi_R = 0.6$, $B_C = B_H = 0.1$ m^2/kW, $T_H = 800$ K, $T_C = 300$ K, $T_S = 200$ K).

Constant ϕ_E,ϕ_R HDHP

Figure 3 shows a plot of M/M_S contours vs T_1 and T_2 for nominal values of the parameters in Table 2. The optimum value of M/M_S is seen to be 0.71 at a lift ($T_2 - T_C$) of 100 K and a drop ($T_H - T_1$) of 240 K. Under the artificial and deleterious constraint $T_1 = T_2$, considerably different values are indicated. The lift rises to 280 K, then drops to 320 K, and M/M_S rises to 0.80. For this same value of M/M_S, once the constraint $T_1 = T_2$ is lifted, one could operate with a lift of 60 K and a drop of 130 K, much more practical values. The example shows very clearly two things: 1) an increased weight savings and 2) more importantly, much greater flexibility in operating conditions for an HDHP when T_1 and T_2 are allowed to differ.

The designer can better choose any T_1, T_2 combination within the 0.8 contour in Fig. 3 rather than be restricted to the point where $T_1 = T_2$. The effect of efficiency on the optimum values of T_1 and T_2 and the system weight is shown in Figs. 4

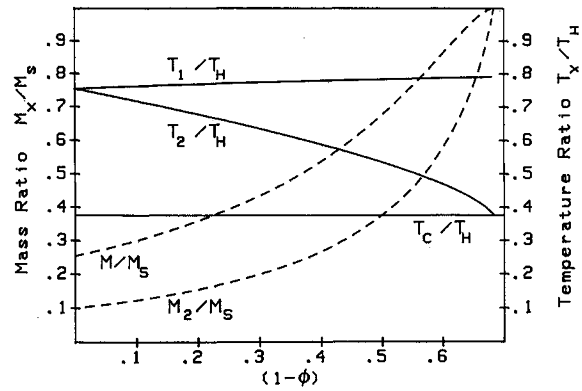

Fig. 4 Optimum conditions, radiator mass dominant ($\phi = \phi_E = \phi_R$, $B_C = B_H = 0$, $T_H = 800$ K, $T_C = 300$ K, $T_S = 200$ K).

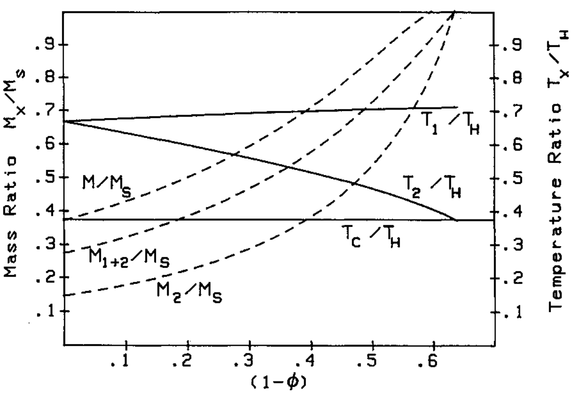

Fig. 5 Optimum conditions, equipment mass included ($\phi = \phi_E = \phi_R$, $B_C = B_H = 0.1$ m^2/kW, $T_H = 800$ K, $T_C = 300$ K, $T_S = 200$ K).

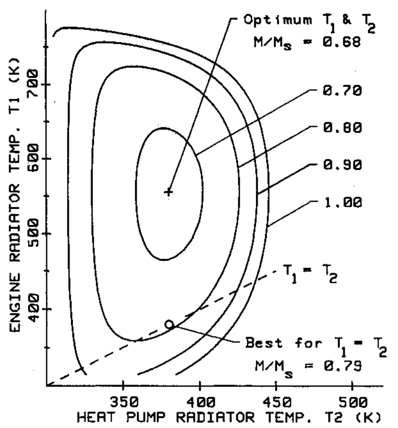

Fig. 6 Mass ratio contours for mercury engine driving isobutane-R113 binary refrigerator ($B_C = B_H = 0.1$ m^2/kW, $\eta_e = 0.8$, $\eta_r = 0$, $T_H = 800$ K, $T_C = 300$ K, $T_S = 200$ K).

and 5. As ϕ_E and ϕ_R approach unity ($1 - \phi$ goes to zero) with $B_C = B_H = 0$ (Fig. 4), the temperatures T_1 and T_2 go to 0.75 T_H as derived by Kerrebrock,[3] and M/M_S goes to 0.25. However, for nonideal (real) equipment, T_1 should go up slightly and T_2 down considerably. Figure 5 shows the effect of nonzero B_C and B_H. In this more realistic case, the optimum value of T_1/T_H falls below 0.75, and the mass ratio M/M_S rises.

Rankine Cycle HDHP

Figure 6 shows the contour plot for a nominal Rankine system (see Table 2). The optimum value of M/M_S is 0.68 at a lift of 75 K and a drop of 250 K. Note how the contours are compressed in T_2 compared to Fig. 3. Under the constraint $T_1 = T_2$, the lift is about unchanged, but the drop goes to 420 K, and M/M_S rises to 0.79. Again, at the same value of M/M_S one has much more attractive choices along the upper left shoulder of the contour. Again the designer has the flexibility of choosing any point within the contour rather than the one dictated by $T_1 = T_2$.

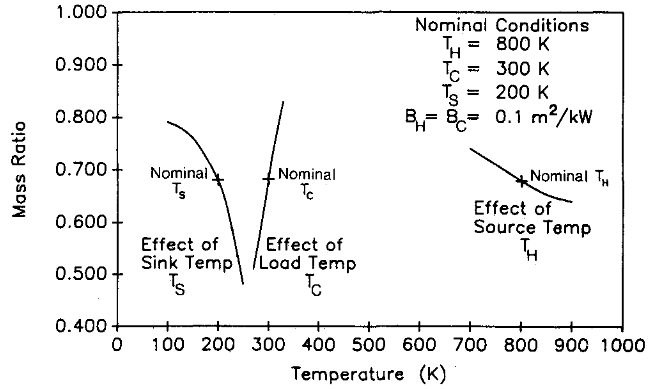

Fig. 7 Sensitivity of mass ratio to service conditions ($B_C = B_H = 0.1$ m^2/kW, $\eta_e = 0.8$, $\eta_r = 0$).

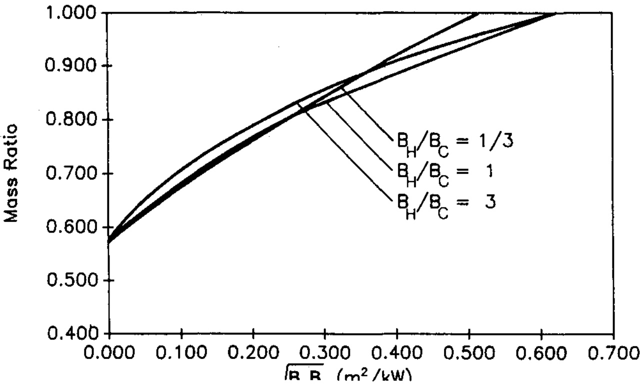

Fig. 8 Effect of equipment mass on mass ratio ($\eta_e = 0.8$, $\eta_r = 0$, $T_H = 800$ K, $T_C = 300$ K, $T_S = 200$ K).

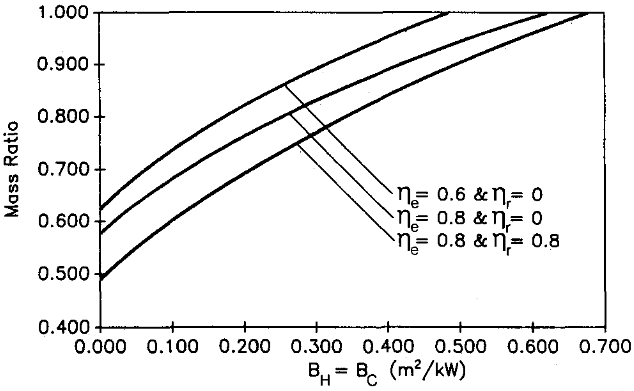

Fig. 9 Effect of expander efficiencies on mass ratio ($T_H = 800$ K, $T_C = 300$ K, $T_S = 200$ K)

The effects of other than the conditions selected as nominal are shown in Fig. 7. An increase in sink temperature or a low required load temperature makes the M/M_S given by a heat pump system look very attractive. Increasing T_H is also beneficial but with diminishing returns.

The effect of the equipment weight penalty is shown in Fig. 8. The product of B_C and B_H and not the individual values separately appears most important in the range investigated.

The effect of isentropic efficiency of the engine expander or the refrigerator compressor is shown in Fig. 9. The importance of efficient equipment is seen.

Discussion

The validity of an optimization using the simple linear Eq. (6) may be questioned. It is thought that fitting a linear rela-

Table 3 Comparison of optima for linear and nonlinear $M_{HP} - Q_H$ relations [$C_1 = \sqrt{B_C B_H}/2$, $C_2 = B_H/2$, $C_3 = B_C$, $C_4 = B_H^2/2B_C$ in Eqs. (19–20)][a]

	Economy of scale model		Penalty of scale model	
	Exact	Linearized	Exact	Linearized
M/M_S	0.68	0.68	0.77	0.78
$T_1\,(K)$	575	574	515	533
$T_2\,(K)$	430	429	375	395

[a]See Table 2 for nominal values.

tion to a more realistic nonlinear model is appropriate for this sort of system analysis. Two nonlinear behaviors can also be imagined. An "economy of scale" (EOS) model could be, for example,

$$M_{HP}^* = \frac{M_{HP}}{aQ_C} = C_1 Q_H^{*1/2} + C_2 Q_H^* + C_3 \tag{19}$$

This model is quite similar to that of Dexter and Haskin.[2] In this model the weight of equipment increases less than proportionally with capacity. A "penalty of size" (POS) model would be, e.g.,

$$M_{HP}^* = \frac{M_{HP}}{aQ_C} = C_4 Q_H^{*2} + C_2 Q_H^* + C_3 \tag{20}$$

In this case the weight of the equipment rises more than proportionally with capacity due to, say, piping and pressure containment. A linearized fit of either of these models can be made by setting an equivalent B_H^* to $\mathrm{d}M_{HP}^*/\mathrm{d}Q_H^*$ and fixing an equivalent B_C^* to match M_{HP}^* at Q_H^*, thus matching a point and slope. The linearized fit is then

$$M_{HP}^* = B_H^* Q_H^* + B_C^* \tag{21}$$

For "exact" results, contours can be computed simply by replacing the linear term $(B_H^* Q_H^* + B_C^*)$ in Eq. (12) by Eq. (18) or (19). Such was done, and linearized results were obtained by iteratively adjusting the match point to the optimum Q_H^* indicated by the linear theory. Table 3 shows the comparisons. They indicate very little difference in the optimum figure of merit M/M_S. The contours in the vicinity of the optimum change only somewhat, a bit more so in the POS model. The main conclusion that T_1 and T_2 are distinctly different when the equipment is nonideal, and its mass considerable, remains in force, regardless of whether a linearized or nonlinear model is used for equipment mass.

Summary and Conclusions

When real, nonideal, equipment is operated, Kerrebrock's rule of rejecting heat at three-quarters of the high temperature heat source requires modification. The rule does apply fairly well to the high temperature T_1, the refrigerant or engine condenser temperature, but not to T_2. Temperature T_2 at which the pumped heat is rejected is found to be 50–80 K above the load temperature, much below 0.75 T_H. By allowing two heat rejection temperatures, rather than imposing $T_1 = T_2$, the designer can improve performance of an HDHP considerably. More importantly, the designer has much greater freedom in designing the process equipment when T_1 and T_2 are chosen separately.

Two parameters B_C and B_H were introduced to characterize the mass of heat pump equipment relative to radiator mass. It was shown that $\sqrt{B_C B_H}$ approximately correlated the mass ratio of the optimized system. High source temperature T_H and highly efficient equipment are needed to make HDHP systems attractive, and their attractiveness is enhanced by high sink temperature T_S and low load temperature T_C.

Acknowledgments

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